Lab 3d

Steady-State Analysis with Uniform Internal Heat Generation

Recall: The general heat transfer equation (for an isotropic material) is:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t} \qquad \text{Eq. 1}$$

If the heat transfer is *one-dimensional* (e.g., in the *x*-direction), and under *steady-state* conditions, Eq.1 reduces to

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)+\dot{q}=0 \qquad \text{Eq. 2}$$

which could be rearranged, assuming constant thermal conductivity (k) and heat generation($\dot{q}$)

$$\frac{d^2T}{dx^2}=-\frac{\dot{q}}{k} \qquad \text{Eq. 3}$$

The solution for this second order differential equation is

$$T(x)=-\frac{\dot{q}x^2}{2k}+c_1x+c_2 \qquad \text{Eq. 4}$$

Boundary conditions:

$$T(x=0)=T_1 \qquad \text{Eq. 5}$$

$$T\ (x=L)=T_2 \qquad \text{Eq. 6}$$

Applying boundary conditions:

$$T=T_1-\frac{x}{L}(T_1-T_2)+\frac{\dot{q}L^2}{2k}\ [(\frac{x}{L}-\left(\frac{x}{L}\right)^2] \qquad \text{Eq. 7}$$

In this lab we want to compare analytical solution that could be obtained using Eq.7 with Abaqus solution:

Consider steady-state conditions for one-dimensional conduction in a plane glass having a thermal conductivity k =0.5 W/m/C and a thickness L= 0.08 m, with constant internal heat generation 700.4 W/m^3 (Figure L3d–1). Determine the temperature distribution.

The units used in this model are SI (kg, m, s, N, ºC).

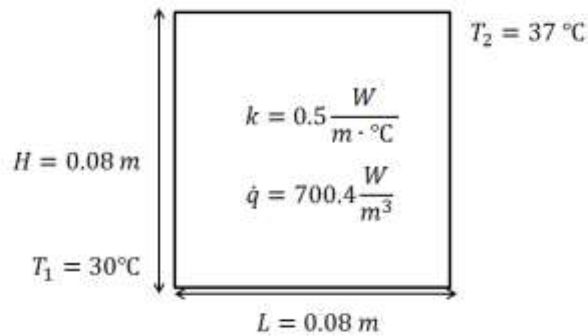

Figure L3d–1. The plate model with an internal heat generation

Create the model as indicated below:

- Create a two-dimensional, deformable body with a planar shell base feature
- Create a linear material with a conductivity = 0.5 W/m/°C.
- Create and assign a solid section to the plate using the linear material.
- Create a single steady-state heat transfer step.
- Assign a fixed temperature = 30 °C and 37 °C to the left and right edges of the plate, respectively.
- Define uniform internal heat flux = 700.4 W/m^3 (Tip: **Load→Create→Body heat flux** (make sure that you define this thermal load for the heat transfer step and not the initial step)
- Change the element type to DC2D4.
- Create and run the job.